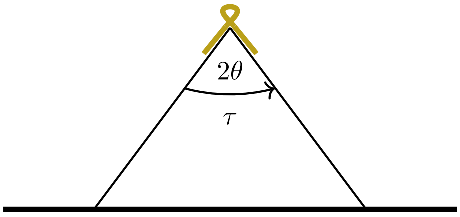
T1: Jumper (10 pts)

Two uniform rods, each of mass m and length ℓ, are connected at one end by a hinge equipped with a torsional spring. The spring exerts equal and opposite restoring torques τ on the rods about the hinge,

$$\tau = 2k\theta,$$

where 2θ is the angle between the rods (in radians), and k ($k \gg mg\ell$) is the torsional spring constant.

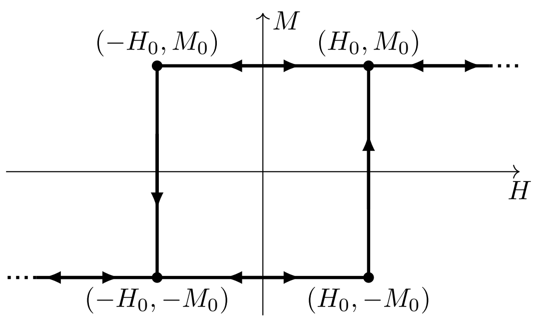


The system is placed on a flat horizontal surface with the free ends of the rods resting on the floor and the hinge above them. The hinge is then pushed downward until the rods lie flat on the floor in opposite directions, corresponding to $\theta = \pi/2$. Friction between the rods and the floor is negligible.

Jumper jumps after the hinge is released. Determine the maximum height h reached by the center of mass of the system to a precision better than 1%.

T2: Hysteresis (10 pts)

Consider a solenoid with $N \gg 1$ turns, radius R and length $\ell \gg R$. The solenoid has a ferromagnetic core with hysteresis curve as shown in the picture. M denotes the magnetization of the core and would be zero without core material. It is related to H, the magnetic field strength generated by the current in the solenoid coil, and B by $B = \mu_0 (H + M)$. M_0 and H_0 are of the same order of magnitude. Assume that M can only change if $|H| \geq H_0$.



An ideal capacitor with capacitance C is now connected to the solenoid forming a closed circuit. Assume that all wires have negligible resistance.

a) **(3.0 pts)** Initially there is a current I_i flowing through the circuit and the capacitor is uncharged. I_i is large enough that $H(I_i) \gg H_0$. After one oscillation the current again reaches a maximum value. Determine the difference between I_i and this value.

b) **(2.3 pts)** Find the maximum current that can be attained after many oscillations.

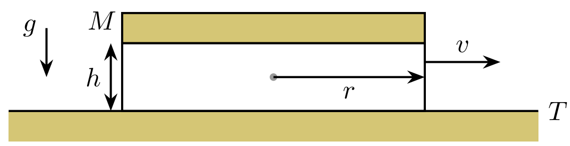
c) **(4.7 pts)** The current $I(t)$ exhibits two qualitatively distinct phases of behaviour, A and B. The system can remain in phase A indefinitely, whereas any single interval spent in phase B has bounded duration. Find the maximal possible duration of a phase-B interval, if I_i is chosen optimally.

T3: Dry ice hockey (10 pts)

At pressure $p_0 = 100\,\text{kPa}$, solid CO_2 (dry ice) sublimates (goes from solid to gaseous state) at $T_s = -78.5\,°\text{C}$. Its saturated vapour pressure follows the Clausius–Clapeyron relation:

$$\frac{\mathrm{d}p_{\text{sat}}}{\mathrm{d}T} = \frac{\mu\lambda p_{\text{sat}}}{RT^2},$$

where the latent heat of sublimation is $\lambda = 600\,\text{kJ/kg}$, the molar mass is $\mu = 0.044\,\text{kg/mol}$, and the gas constant is $R = 8.3\,\text{J/(mol}\cdot\text{K)}$. The thermal conductivity of the CO_2 gas is $\kappa = 10\,\text{mW/(m}\cdot\text{K)}$ and its dynamic viscosity is $\eta = 10\,\mu\text{Pa}\cdot\text{s}$. The density of dry ice is $\rho = 1500\,\text{kg/m}^3$ and the gravitational acceleration is $g = 10\,\text{m/s}^2$.



A puck of radius $r = 10\,\text{mm}$ consists of a disc of dry ice of thickness $h = 1\,\text{mm}$, and a metal disc of mass $M = 0.01\,\text{kg}$ on top of it. The initial temperature of the puck is T_s and the ambient pressure is p_0.

The puck is placed on a horizontal metal plate which is held at a constant temperature $T = T_s + \Delta T$, and given an initial horizontal velocity $v = 10\,\text{mm/s}$. After a very long time, the horizontal displacement L of the metal disc is measured.

a) **(2 pts)** When ΔT is sufficiently small, L is negligible and independent of ΔT. However, when ΔT reaches a critical value ΔT_c, the function $L(\Delta T)$ starts to grow. Estimate ΔT_c.

b) **(8 pts)** Estimate the maximal value L_{max} of the function $L(\Delta T)$.

Treat the CO_2 gas as ideal. Assume no tilting of the puck at any moment, all surfaces are perfectly smooth, and the thermal conductivities of the metal, dry ice, and gas satisfy $\kappa_{\text{metal}} \gg \kappa_{\text{ice}} \gg \kappa$.