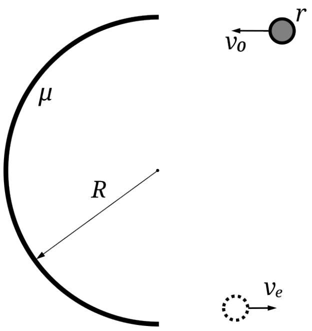
T1: Sliding puck (10 pts)

A puck (a small disc) with radius r and uniform density is moving on a horizontal plane with the velocity v_0 without rotation. The puck meets a fixed half-circular wall with a radius $R \gg r$ and starts to move along the wall. The coefficient of friction with the wall is μ, and friction with the horizontal plane is negligible.



a) (8 pts) Find the velocity of the puck v_e when it leaves the wall.

b) (2 pts) Sketch the graph $v_e(\mu)$. Indicate important features of the graph. You are encouraged to sketch the graph even if you haven't found the exact formula for v_e.

T2: Spaceships (10 pts)

Alice and Bob are twin astronauts on a long space mission. After many years, they are finally approaching each other to reunite. Alice's spaceship is moving towards Bob's spaceship at a speed of $u = \frac{3}{5}c$, where c is the speed of light.

During their approach, both Alice and Bob send gifts to each other. Alice sends gifts to Bob at regular time intervals Δt_0 in her own frame of reference, with each gift travelling at a velocity $v = \frac{4}{5}c$ (again, in her frame of reference). Similarly, Bob sends gifts to Alice at the same regular time intervals Δt_0 in his own frame of reference, with each gift also travelling at a velocity $v = \frac{4}{5}c$ in his frame of reference. Assume that the distance L between Alice and Bob is so large that there are many gifts in transit at any given moment.

a) (5 pts) In Bob's reference frame, find

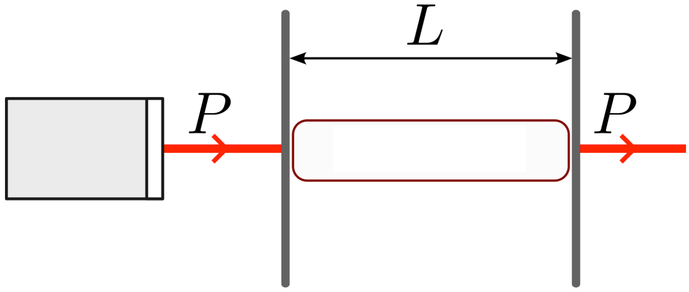
   (i) the distance between two successive gifts sent by Alice, and

   (ii) the time interval Δt_1 at which these gifts from Alice arrive at Bob's spaceship.

b) (5 pts) At a given instant, Alice can see a number of gifts moving away from her and a number of gifts moving towards her. What is the ratio between these two numbers?

T3: Fabry–Pérot interferometer (10 pts)

A Fabry–Pérot interferometer consists of two identical parallel planar mirrors separated by a distance L. The space between and outside the mirrors is filled with air. The mirrors are partially reflective; when light is aimed towards one of these mirrors along the normal direction, the reflected beam has intensity $R < 1$ times the intensity of the incident beam. Assume that the mirrors are symmetric, meaning they interact the same way with light incident from either side, and lossless. Assume also that they are highly reflective, meaning $1 - R \ll 1$. A monochromatic laser beam of power P is aimed towards the interferometer perpendicular to the mirrors. The distance L is chosen so that the back-reflected beam vanishes, i.e. all the optical power is transmitted through the interferometer.



a) (3pts) Show that the laser beam must acquire a nonzero phase shift ϕ when it passes through either of the mirrors.

b) (2pts) Find the magnitude of ϕ.

c) (4pts) At a certain moment, the incident laser beam is switched off rapidly. Find the total energy of the light that travels back from the interferometer towards the laser after the laser is switched off.

d) (1pt) Estimate the duration of the light pulse that travels back towards the laser.