SVN-S1 T1

Cover Sheet for Solutions

7. EuPhO 23

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T1. ~~[illegible]~~

$\gamma = \frac{da}{dT}$

$a = 15,0\,mm$

$b = 0,2\,mm$

$A = 0,1$

$k = 0,3\,\frac{W}{m \cdot K}$

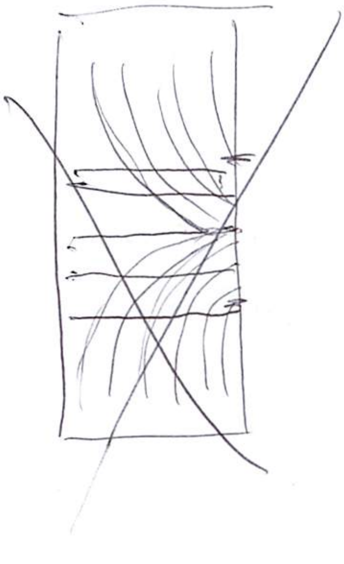
$\gamma = 2,5 \cdot 10^{-4}\,K^{-1}$

$T_a = 20°C$

$\sigma = 0,5\,mm$

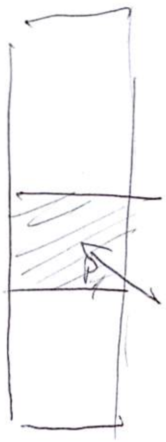
$P_L = 20\,mW = 2 \cdot 10^{-2}\,W$

Can you say that the heat is distributed equally across z-axis ?



If it is approximately equal across the entire cross-section, then :

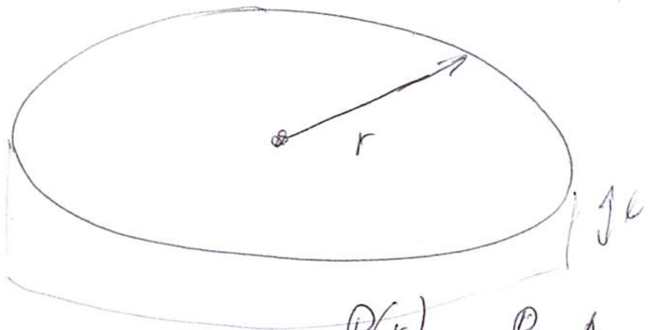
T1.



$j \cdot \pi \sigma^2 = P_L$

$j \cdot \pi \sigma^2 \cdot A = P_1$

inside ~~σ~~ $r \le \sigma$:



$P(r) = P_L \cdot A \cdot \frac{r^2}{\sigma^2}$

$kA \frac{dT}{dr}$

which has to go through the ~~external~~ side at r

$P(r) = -k \cdot 2\pi r \cdot b \cdot \frac{dT}{dr}$

$P(r) = P_L \cdot A \cdot \frac{r^2}{\sigma^2} = -k \cdot 2\pi r \cdot b \cdot \frac{dT}{dr}$

$$\frac{P_L A r}{\sigma^2 \cdot k \cdot 2\pi \cdot b} = -\frac{dT}{dr}$$

$coeff \cdot r\,dr = dT$

$coeff \frac{r^2}{2}\Big|_{\sigma}^{0} = -T\Big|_{T_{edge}}^{T_{center}}$

Tc.

$$\text{coeff} \cdot \frac{\sigma^2}{2} = T_{center} - T_{edge}$$

or

$$\text{coeff} \left.\frac{r^2}{2}\right|_{\sigma}^{r} = -T \Big|_{T_{edge}}^{T(r)}$$

$$\text{coeff} \cdot \left(\frac{\sigma^2}{2} - \frac{r^2}{2}\right) = T(r) - T_{edge}$$

now ~~at the~~ outside of σ:

$$A \cdot P_L = P(r) = const = -k \cdot 2\pi r \cdot b \cdot \frac{dT}{dr}$$

$$A \cdot P_L = -k \cdot 2\pi r b \cdot \frac{dT}{dr}$$

$$\frac{A \cdot P_L}{2\pi r}$$

$$\boxed{\frac{A \cdot P_L}{r} \cdot dr = -k \cdot 2\pi \cdot b \cdot dT}$$

$$A \cdot P_L \cdot \ln(r)\Big|_{a}^{r} = -k \cdot 2\pi b \left(\cdot T\right)\Big|_{T_L}^{T(r)}$$

$$A \cdot P_L \cdot \ln\left(\frac{a}{r}\right) = k \cdot 2\pi b \cdot \left(T(r) - T_L\right)$$

So at ~~the~~ $r = \sigma$, $T(r) = T_{edge}$:

$$A \cdot P_L \cdot \ln\left(\frac{a}{r}\right) = k \cdot 2\pi b \cdot \left(T(r) - T_L\right)$$

$$\frac{A P_L}{k \cdot 2\pi b} \ln\left(\frac{a}{r}\right) = T(r) - T_L$$

$$T_L + \frac{A P_L}{k \cdot 2\pi b} \cdot \ln\left(\frac{a}{r}\right) = T(r)$$

T1. outside: $r > \sigma$ $T(r) = T_h + \frac{AP_L}{k \cdot 2\pi b} \cdot \ln\left(\frac{a}{r}\right)$

$$T_{edge} = T_h + \frac{AP_L}{k \cdot 2\pi b} \cdot \ln\left(\frac{a}{\sigma}\right)$$

→ going back to the region $r \leq \sigma$:

$$\int_0^r \frac{P_L A}{\sigma^2 \cdot k \cdot 2\pi b} \cdot r\,dr = \int_{T_{edge}}^{T(r)} -dT$$

$$\frac{P_L A}{\sigma^2 k 2\pi b} \cdot \left(\frac{\sigma^2}{2} - \frac{r^2}{2}\right) = T(r) - T_{edge}$$

$$\frac{P_L A}{k \cdot 2\pi b} \cdot \left(\frac{1}{2} - \frac{r^2}{2\sigma^2}\right) = T(r) - T_{edge}$$

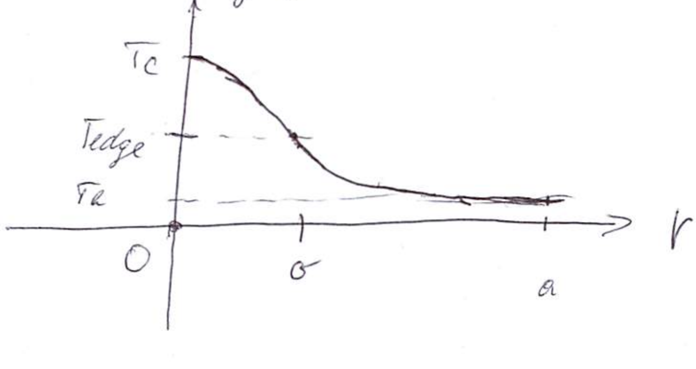
$$T(r) = T_h + \frac{AP_L}{k \cdot 2\pi b}\left(\frac{1}{2} - \frac{r^2}{2\sigma^2} + \ln\left(\frac{a}{\sigma}\right)\right)$$

at $r = 0$: $T(0) = T_{center}$

$$T_{center} = T_h + \frac{AP_L}{k \cdot 2\pi b}\left(\frac{1}{2} + \ln\left(\frac{a}{\sigma}\right)\right)$$

TO DO: check edge $-\ln(r)$ T_h

b) a) Qualitative graph:



T1. b) in the ~~d~~ vicinity of the outer of the disk:

$$\int \frac{P_L A r}{\sigma^2 \cdot k \cdot 2\pi b} \cdot dr = -\int dT$$

$$\frac{P_L A \cdot r^2}{\sigma^2} = -2\pi r b \cdot \frac{dT}{dr} \cdot k$$

$$\frac{P_L A r\, dr}{2\pi b \sigma^2 k} = -dT$$

~~$\frac{P_L A}{k \cdot 2\pi b\, \sigma^2} \cdot \frac{r^2}{2}\Big|^0$~~

$$\frac{P_L A}{\sigma^2 \cdot k \cdot 2\pi b} \cdot \int_r^0 r\, dr = -\int_{T(r)}^{T_{center}} dT$$

$$\frac{P_L A}{\sigma^2 \cdot k \cdot 2\pi b} \cdot \left(\frac{r^2}{2}\right) = +T_{center} - T(r)$$

$$T(r) = T_{center} - \frac{P_L A}{\sigma^2 \cdot k \cdot 2\pi b} \cdot \frac{r^2}{2}$$

$$m = -\frac{P_L A}{4\pi b \cdot \sigma^2 k} \quad -1{,}06 \cdot 10^{7} \frac{K}{m^2}$$

~~$T_{edge} = T_{center} = T_{edge}$~~ $T_{center} = T_{edge} - m r^2$

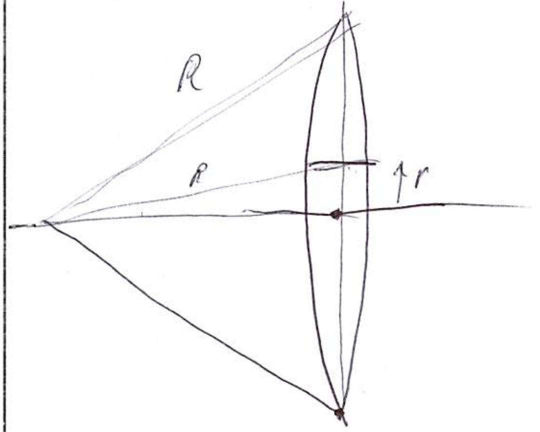
$$T_{center} = T_h + \frac{A P_L}{k \cdot 2\pi b}\left(\frac{1}{2} + \ln\left(\frac{a}{\sigma}\right)\right)$$

$$T_{center} \approx 40{,}7\,°C = T_C$$

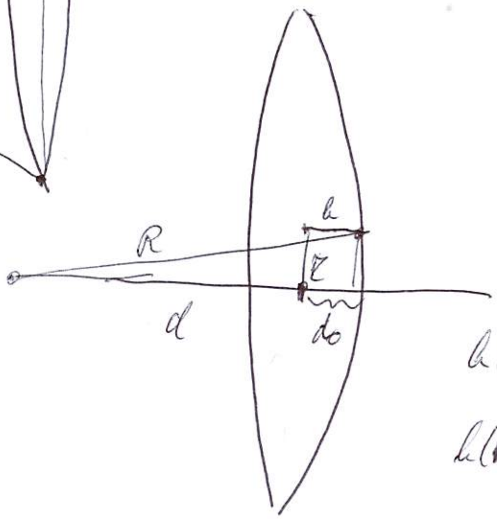
c) Negative gradient needed for the heat to flow outwards

b) $T(r) \approx 40{,}7°C - 1{,}06 \cdot 10^{7} \frac{K}{m^2} \cdot r^2$ for $r < \sigma$

T1. c) Showing that it's focused at one point:
analogy with a lens: (first calculating for lens)



thickness of a lens at r-distance to optical axis:



$d + d_0 = R$

$d = R - d_0$

$h(r) = \sqrt{R^2 - r^2} - d$

for thin lens:

$h(r) = R\left(1 - \frac{r^2}{2R^2}\right) - d =$

$h(r) = h(0) - \frac{r^2}{2R}$

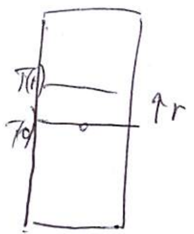
~~So the~~ n in a lens $n = const = n_0$

so optical thickness is: $h(r) \cdot n_0 = h(0) n_0 - \frac{r^2}{2R} \cdot n_0$

for a thermal lens $\delta = \frac{dn}{dT}$, thus $\Delta n = \delta \cdot \Delta T$

the thickness is equal everywhere, but n changes:

optical thickness is:



analogous

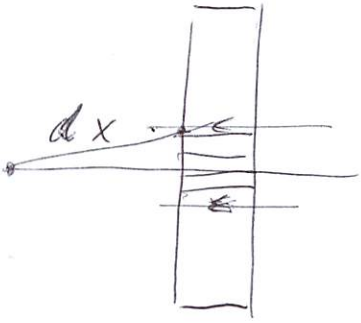
$b \cdot n_{lens,\,center} + b \cdot \Delta n =$

$= b \cdot n_{lens,\,center} + b \cdot \delta (T(r) - T_C) =$

$= b \cdot n_{lens,\,center} + b\,\delta \cdot (-\alpha r^2)$ which is the same formula

T1. c) f – ?

f of a lens ~~$\frac{1}{f}$ …~~



Using Fermat's principle, we can conclude that ~~these~~ parallel wavefront arrives at the same time at the focus:

~~f:~~ x is the distance ~~the~~ a ray travels to the focus:

$$\underline{x^2 = f^2 + r^2} \text{ for } r \ll f$$

time a ray at r needs to get to focus: $x \approx f + \frac{r^2}{2f}$

$$t = \frac{x}{c} \cdot n_{AIR} + \frac{6n_c + 6\delta(1 + m r^2)}{c}$$

must be equal:

$$t = \frac{f}{c} \cdot n_{AIR} + \frac{6n_c}{c}$$

=>

$$\underline{\frac{r^2}{2f \cdot c}} = \underline{-\frac{6\delta}{c} m r^2}$$

$$\frac{1}{2f} = -6\delta m$$

$$f = \frac{1}{-2\delta 6 m}$$

$f \approx 0{,}94\,m$ => $r \ll f$ is reasonable