Set 117

SGP-S2 E1

Cover Sheet for Solutions

7. EuPhO 23

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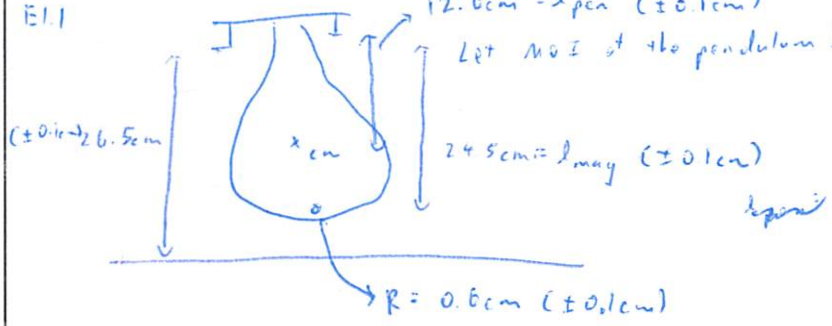
E1.1



Let MoI of the pendulum be I.

For any physical pendulum, $I\ddot{\theta} = -mg\,l_{cm}\sin\theta$

$\approx -mg\,l_{cm}\,\theta$

reaction time × 2

$\omega = \sqrt{\frac{mg\,l_{cm}}{I}}$

without magnets: $T_1 = \frac{87.10 \pm 0.2}{100} = (0.8710 \pm 0.002)\,s = \sqrt{\frac{M_{pen}\,g\,l_{pen}}{I_{pen}}}$

with magnets: $T_2 = \frac{89.63 \pm 0.2}{100} = (0.8963 \pm 0.002)\,s = \sqrt{\frac{M_{pen}\,g\,l_{pen} + M_{mag}\,g\,l_{mag}}{I_{pen} + M_{mag}(l_{mag}^2 + \frac{1}{2}R^2)}}$

unknowns: M_{pen}

M_{mag}

I_{pen}

From T_1, $I_{pen} = \frac{M_{pen}\,g\,l_{pen}}{T_1^2} \Rightarrow T_2 = \sqrt{\frac{M_{pen}\,g\,l_{pen} + M_{mag}\,g\,l_{mag}}{M_{pen}\,g\,l_{pen}/T_1^2 + M_{mag}(l_{mag}^2 + \frac{1}{2}R^2)}}$

$\frac{T_2^2}{T_1^2}\,g\,l_{pen}\,M_{pen} + T_2^2(l_{mag}^2 + \frac{1}{2}R^2)\,M_{mag} = g\,l_{pen}\,M_{pen} + g\,l_{mag}\,M_{mag}$ (±1.7%, ±1.3%, ±0.06%, ±0.4%)

$M_{mag} + M_{pen} = M_{tot}$

M_{pen} = (50.4 ± 1.008) g

M_{mag} = (1.95 ± 1.008) g

E1.2 (a) ~~d: 28.0 cm~~

d (cm)	T (s)	ω_{mag} (rad s^{-1})	ln d	ln ω_{mag}
~~8.0 ± 0.1~~	~~$\frac{64.75}{50}$ = 1.295 ± 0.004~~			
9.0 ± 0.1	$\frac{50.22}{50}$ = 1.004 ± 0.004 ~~1.447 ± 0.004~~	5.814 ± 0.0254	-2.408 ~~-2.147~~	1.760
10.0 ± 0.1	$\frac{56.56}{50}$ = 1.131 ± 0.004	4.275 ± 0.0511	-2.303 ~~2.303~~	1.453 ✓
11.0 ± 0.1	$\frac{50.19}{50}$ = 1.0038 ± 0.004	3.156 ± 0.0841	-2.207 ~~2.394~~	1.149 ✓
12.0 ± 0.1	$\frac{48.09}{50}$ = 0.9618 ± 0.004	2.543 ± 0.1134	-2.120 ~~2.483~~	0.933 ✓
14.0 ± 0.1	$\frac{46.57}{50}$ = 0.9314 ± 0.004	1.906 ± 0.160	-1.966 ~~2.639~~	0.643 ✓
18.0 ± 0.1	$\frac{45.22}{50}$ = 0.9044 ± 0.004	0.936 ± 0.345	-1.715 ~~2.890~~	-0.0661
9.5 ± 0.1	~~$\frac{50.14}{50}$ = 1.0028 ± 0.004~~ $\frac{63.68}{50}$ = 1.2736 ± 0.004	4.950 ± 0.0373	-2.354 ~~2.251~~	1.605 ✓

$$\omega^2 = \omega_1^2 - \omega_{mag}^2 \Rightarrow \omega_{mag} = \sqrt{\omega_1^2 - \omega^2}$$

~~$\Delta\omega$~~

$$\omega_1 = (7.010 \pm 0.0156)\,\text{rad s}^{-1}$$

$$\frac{\Delta\omega_{mag}}{\omega_{mag}} = \frac{\Delta(\omega_1^2 - \omega^2)}{2(\omega_1^2 - \omega^2)}$$

$$= \frac{\omega_1 \Delta\omega_1 + \omega_2 \Delta\omega_2}{\omega_1^2 - \omega^2}$$

$$\frac{\Delta\omega}{\omega} = \frac{\Delta T}{T} \Rightarrow \Delta\omega = \frac{\Delta T}{T}\omega$$

$$\omega_{mag}^2 = \frac{6\mu_0}{I\pi} \cdot j_1 \cdot j_2 \cdot \frac{l^2}{d^5} \Rightarrow \ln\omega_{mag} = \frac{\ln\left(\frac{6\mu_0}{I\pi} \cdot j_1 \cdot j_2 \cdot l^2\right)}{2} - \frac{5\ln d}{2}$$

$m = \frac{0.2 - 1.3}{-1.86 + 2.29}$

$= -2.444$ very close to $-\frac{5}{2}$ yay ☺

$0.2 = -2.444(-1.86) + c \Rightarrow c = ~~4.7466...~~ -4.3458...$

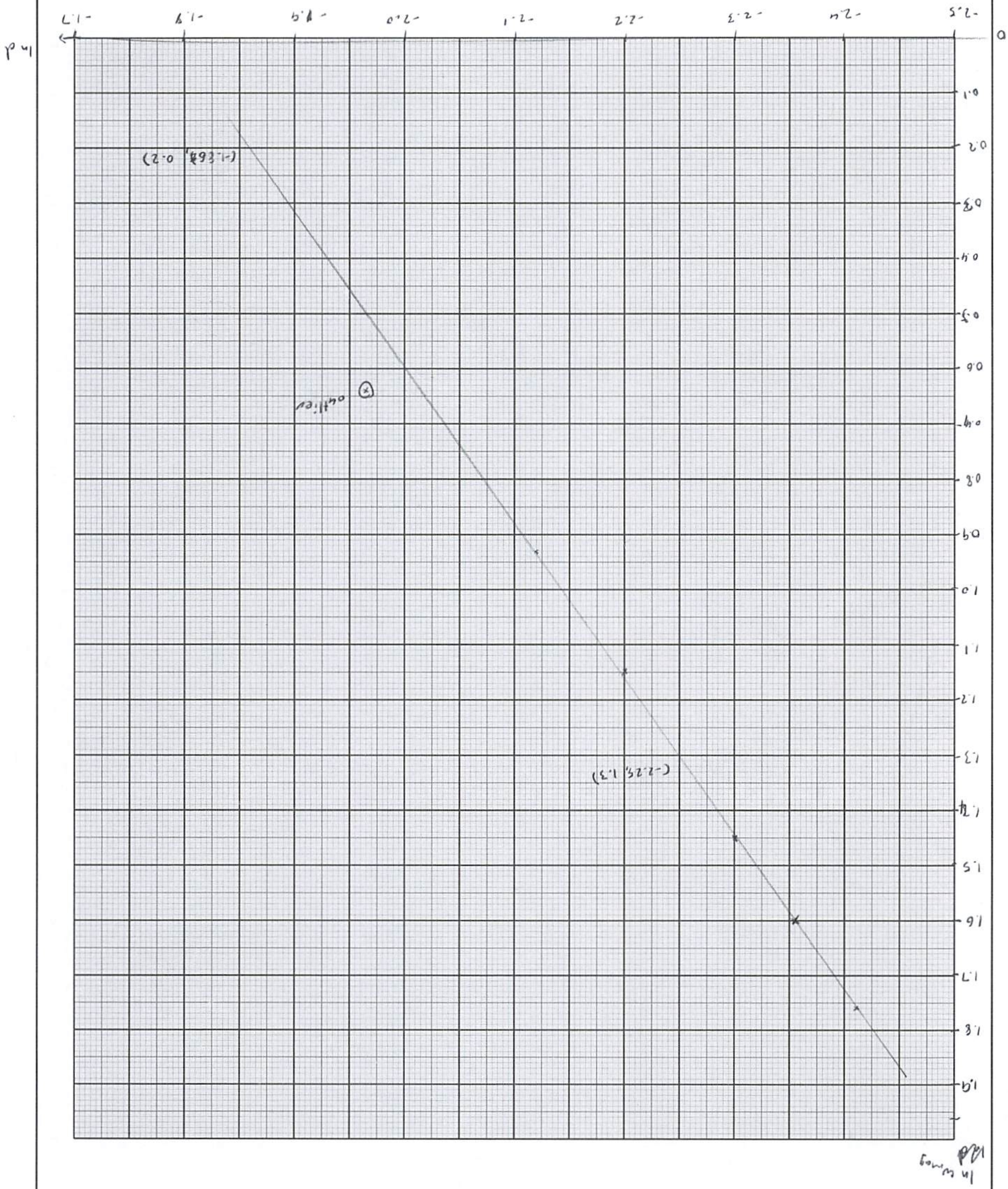
$= \frac{1}{2}\ln\left(\frac{6\mu_0}{I\pi} \cdot j_1 \cdot j_2 \cdot l^2\right)$

$= \frac{1}{2}\ln\left(\frac{6\mu_0}{I\pi} \cdot 2.4 j_1^2 \cdot l^2\right) \Rightarrow j_1 = ~~5.614~~ 6.316\ Am^2$

$I = 0.08212\ kgm^2$, $l = 24.5\ cm$

For the pendulum magnets, ~~$\frac{5.614}{0.00195} = 2.88 \times 10^3\ Am^2 kg^{-1}$~~ $\frac{6.316}{0.00195} = 3239\ Am^2 kg^{-1}$

~~2.4 × 5614~~



E1.2

E1.3 (a)

d (cm)	T (s)	ω_{mag} (rad s^{-1})	ln d	ln ω_{mag}
6.5	$\frac{39.82}{50}$ = 0.7964 ± 0.004	~~6.204~~ 3.620	-2.733	1.286
6.0	$\frac{36.78}{50}$ = 0.7356 ± 0.004	~~6.126~~ 4.880	-2.813	1.585
5.5	$\frac{33.47}{50}$ = 0.6694 ± 0.004	5.~~926~~ 6.242	-2.900	1.831
7.0	$\frac{40.75}{50}$ = 0.815 ± 0.004	3.208	-2.659	1.166
8.0	$\frac{42.66}{50}$ = 0.8512 ± 0.004	2.312	-2.526	0.838
5.0	$\frac{27.72}{50}$ = 0.5544 ± 0.004	8.905	~~-3.000~~ ~~-2.7~~ -2.996	2.187

$\omega^2 = \omega_1^2 + \omega_{mag}^2 \Rightarrow \omega_{mag} = \sqrt{\omega^2 - \omega_1^2}$.

$\omega_{mag,F}^2 = kd^{\alpha} \Rightarrow \ln \omega_{mag,F} = \frac{1}{2}\ln k + \frac{\alpha}{2}\ln d$.

(b)

$m = \frac{1.0 - 2.2}{-2.59 + 3.02}$

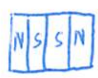
$= -2.79$

≈ -2.8

$= \frac{\alpha}{2} \Rightarrow \alpha =$ ~~-5.6~~ -5.58.

(c)

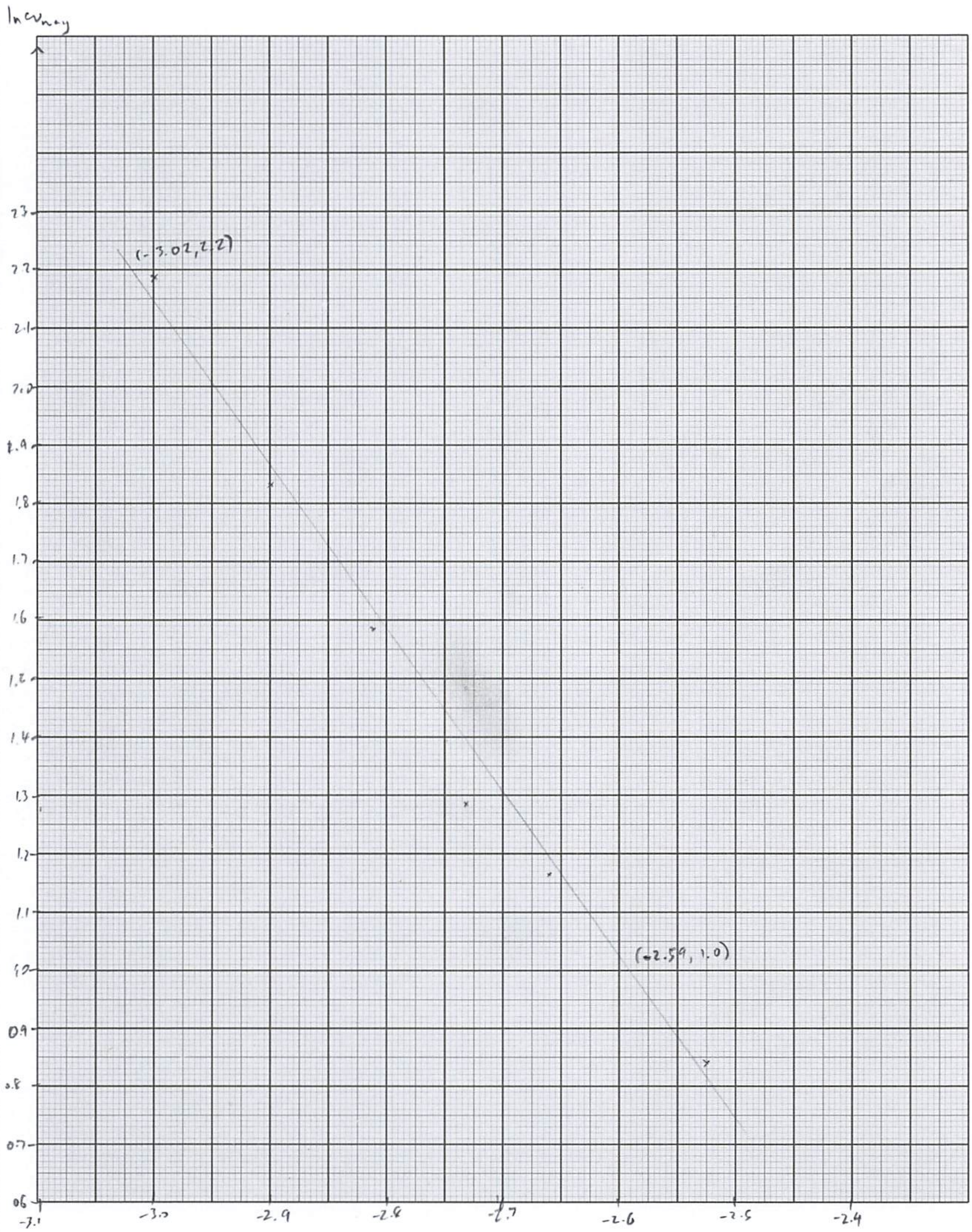
F1



F2

since both ends of F1 have the same polarity and both ends of F2 have the same polarity, but opposite to F1. also result in ~~α≥5~~ $\alpha < -5$ since the magnet ~~dipole~~ behaves more like a quadrupole

E1.3


lnω_max
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
-3.1
-3.0
-2.9
-2.8
-2.7
-2.6
-2.5
-2.4
(-3.02, 2.2)
(-2.59, 1.0)

E1.4 (a) $\omega^2 = \omega_1^2 - \omega_{mag}^2$

$= 0 \Rightarrow \omega_{mag}^2 = \omega_1^2$ ← ~~~~ $\omega_1 = 7.010\ rad\,s^{-1}$

$= \frac{6\mu_0}{I\pi} \cdot j_1 j_2 \cdot \frac{l^2}{d^3}$ ← $l = 24.5\ cm$

↑ $I = 0.08212\ kg\,m^2$

↖ $j_1 =$ ~~~~ $6.316\ Am^2$, $j_2 = 2.4 j_1$

$\therefore d = 0.0807\ m$

$= 8.07\ cm$

(b) → shown onto grid paper

→ must take quickly or damping ∴

A (cm)	T (s)	ln A	ln T
~~1.0~~	~~~~	-1.104	
0.3	$\frac{15.94}{5} = 3.188$	-0.511	1.159
0.6	$\frac{12.25}{5} = 2.45$	0.0953	0.896
1.1	$\frac{8.06}{5} = 1.612$	-0.1054	0.477
0.9	$\frac{9.28}{5} = 1.856$		0.618
0.8	$\frac{11.00}{5} = 2.20$	-0.2231	0.788

Assume ~~~~ $T = kA^{\alpha}$

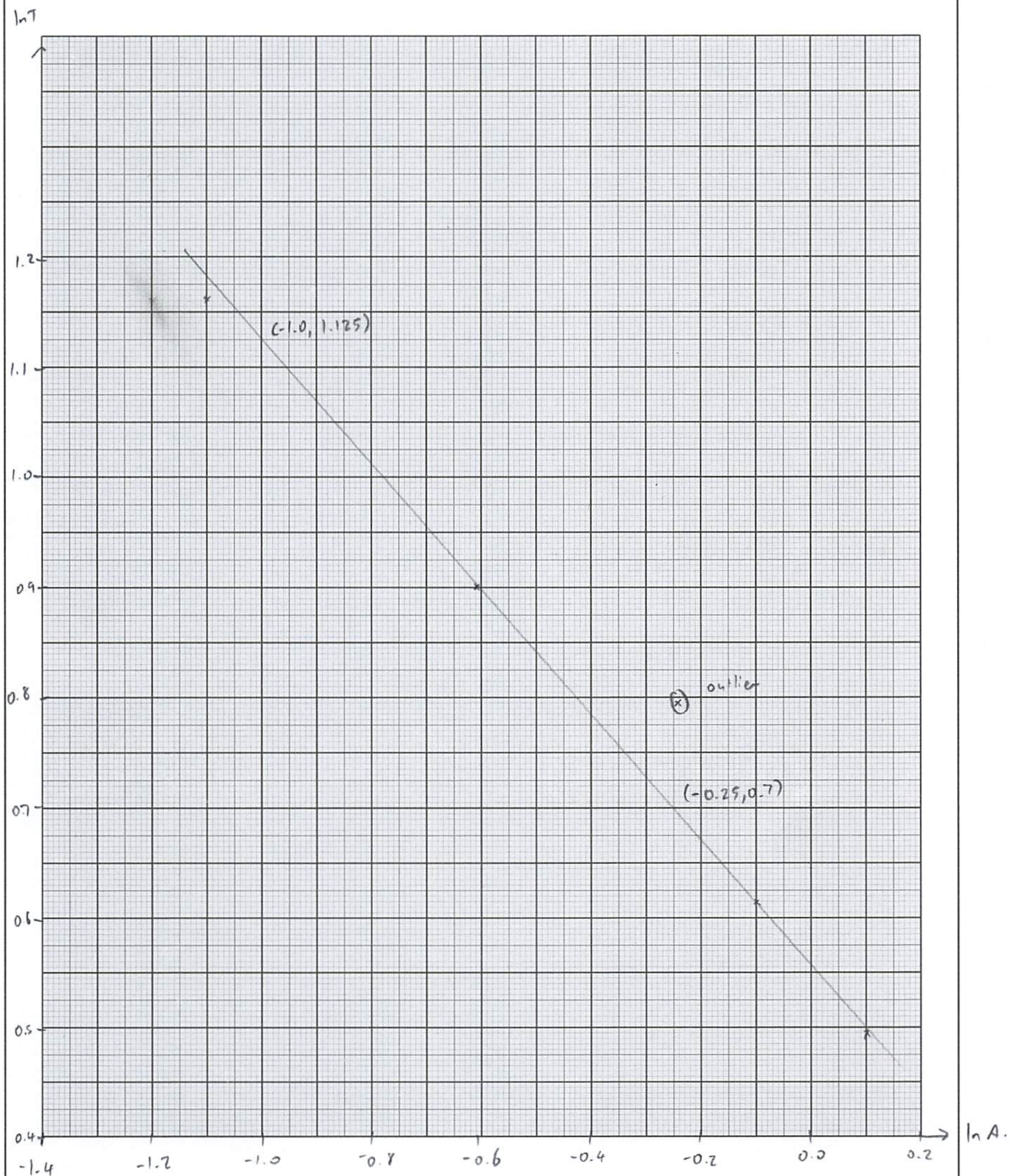
$\ln T = \ln k + \alpha \ln A$

maybe not ~~optimal~~ -0.5 because ~~too~~ ~~tight~~ amplitude is limited to accuracy of the grid paper and the laser beam is quite thick (0.4cm diameter)

$m = \frac{0.7 - 1.125}{-0.25 + 1.0} = -0.566$

≈ -0.5

$T \propto \frac{k}{\sqrt{A}}$? ~~it might not be rational~~


lnT
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
(-1.0, 1.125)
outlier
(-0.25, 0.7)
-1.4
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
0.2
ln A.