ROU-S3 T2

Cover Sheet for Solutions

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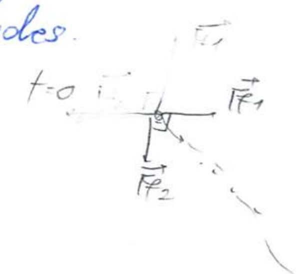
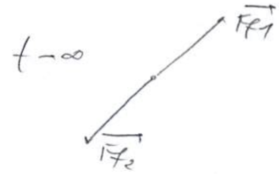
• After a long time, the velocity $\vec{v_\infty}$ of the brick will become stationary. However, the two frictional forces (of equal magnitude, because the ~~normal~~ normal forces and kinetic friction coefficients are the same for both plates) cannot be both 0, as the relative velocity of the block will be nonzero w.r.t. at least one of the plates). The only way in which the block can remain in mechanical equilibrium is if the two frictional forces (and hence the two relative velocity vectors) are exactly opposite and cancel each other, due to their equality of magnitudes.



• Let $\vec{v_\infty}$ be the final velocity.
The required condition is that

$$\vec{v_\infty} - \vec{v_1} = -k\,(\vec{v_\infty} - \vec{v_2}),$$

where $k > 0$ is a scalar. So



$$\vec{v_\infty} + k\vec{v_\infty} = k\vec{v_2} + \vec{v_1}$$

$$\Leftrightarrow \quad \vec{v_\infty} = \frac{1}{k+1}\left(\vec{v_1} + k\vec{v_2}\right), \text{ for some } k > 0. \quad (1)$$

$\vec{N_1} = \vec{N_2}$ (zero gravity)

• Let the velocity of the ~~body~~ block at some point be $\vec{v} = v_x \hat{x} + v_y \hat{y}$. The frictional force vectors ~~along~~ will be

$$\vec{F_{f1}} = -F_{f1} \cdot \frac{\vec{v} - \vec{u_1}}{|\vec{v} - \vec{u_1}|} ; \quad \vec{F_{f2}} = -F_{f2} \cdot \frac{\vec{v} - \vec{u_2}}{|\vec{v} - \vec{u_2}|} .$$

• Since $F_{f1} = F_{f2} = F_f$, ~~we~~ get the total resultant force is

$$\vec{F} = \vec{F_{f1}} + \vec{F_{f2}}$$
$$= -F \left(\frac{\vec{v} - \vec{u_1}}{|\vec{v} - \vec{u_1}|} + \frac{\vec{v} - \vec{u_2}}{|\vec{v} - \vec{u_2}|} \right) .$$

• Let us consider a frame moving with velocity

$$\vec{U} = \frac{u_2 \vec{u_1} + u_1 \vec{u_2}}{u_1 + u_2}$$

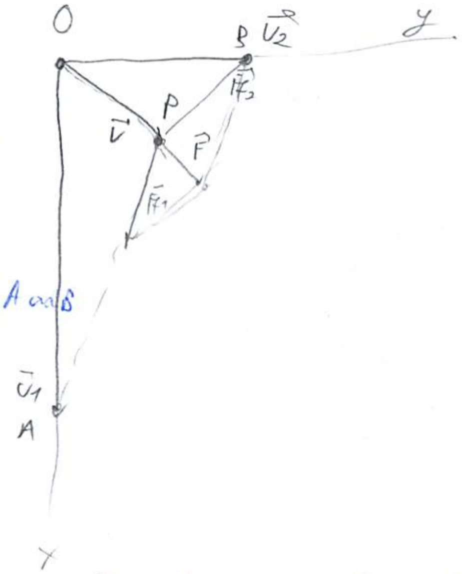
• The relative velocities of the two plates ~~in~~ ~~this f~~ with respect to this frame are, respectively,

$$\vec{u_{1,r}} = \vec{u_1} - \vec{U} = \frac{u_1 \vec{u_1} - u_1 \vec{u_2}}{u_1 + u_2} = \frac{u_1}{u_1 + u_2} (\vec{u_1} - \vec{u_2}) ; \text{ and}$$

$$\vec{u_{2,r}} = \vec{u_2} - \vec{U} = \frac{-u_2 \vec{u_1} + u_2 \vec{u_2}}{u_1 + u_2} = -\frac{u_2}{u_1 + u_2} (\vec{u_1} - \vec{u_2}) .$$

~~• These two velocities are along the same line and have opposite~~

• Let us consider the process in velocity space. At any time, the point P corresponding to the velocity of the brick moves under the action of the two frictional forces, $\vec{F_{f1}}$ and $\vec{F_{f2}}$, the changes due to which are oriented towards the ends of the velocities $\vec{u_1}$ and $\vec{u_2}$, A and B (see fig.). This occurs because the frictional forces are parallel to $\vec{v}-\vec{u_i}$ $(i=1,2)$, where $\vec{v}$ is the velocity of the block. Hence, the point P will move along the direction of $\frac{\vec{v}-\vec{u_1}}{|\vec{v}-\vec{u_1}|}+\frac{\vec{v}-\vec{u_2}}{|\vec{v}-\vec{u_2}|}$.



• Let z_A be the distance from P to A, and z_B, from P to B. We have (since $\vec{F}=m\vec{a}=m\frac{d\vec{v}}{dt}$),

$$\frac{dz_A}{dt}=-\frac{\vec{F}\cdot\hat{PA}}{m}=-\frac{(\vec{F_{f1}}+\vec{F_{f2}})\cdot\frac{\vec{F_{f1}}}{F_{f1}}}{m}=-\frac{F_{f1}^2+\vec{F_{f1}}\vec{F_{f2}}}{mF_{f1}};$$

$$\frac{dz_B}{dt}=-\frac{\vec{F}\cdot\hat{PB}}{m}=-\frac{(\vec{F_{f1}}+\vec{F_{f2}})\cdot\frac{\vec{F_{f2}}}{F_{f2}}}{m}=-\frac{F_{f2}^2+\vec{F_{f1}}\cdot\vec{F_{f2}}}{mF_{f2}},$$

where m is the mass of the brick. Since $F_{f1}=F_{f2}$, these two are equal:

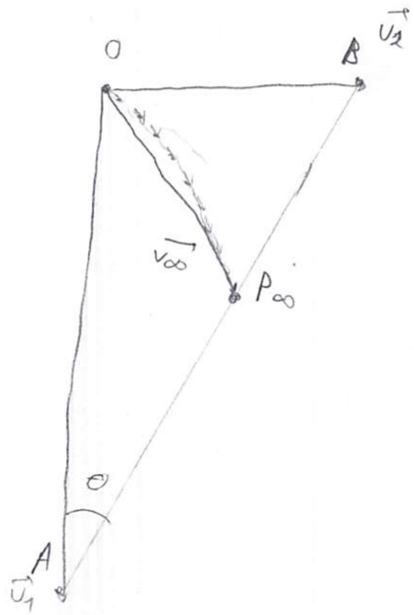
$$\frac{dz_A}{dt}=\frac{dz_B}{dt}\Rightarrow\frac{d}{dt}(z_A-z_B)\Leftrightarrow\boxed{z_A-z_B=\text{const.}}\quad(2)$$

· In the beginning, $z_A = v_1$, $z_B = v_2$. At $t \to \infty$, according to the earlier argument (on page 1), P_∞ must lie between ~~$\vec{v_1}$ and~~ A and B (i.e. $\vec{v_\infty}$ is a linear combination of $\vec{v_1}$ and $\vec{v_2}$, see Fig.). ~~The~~ Using (2),

$$v_1 - v_2 = P_\infty A - P_\infty B$$

~~$P_\infty A^2 + P_\infty B^2 =$~~

$$P_\infty A + P_\infty B = \sqrt{v_1^2 + v_2^2} \quad \Rightarrow$$



$$\Rightarrow \begin{cases} P_\infty A = \dfrac{v_1 - v_2 + \sqrt{v_1^2 + v_2^2}}{2}, \\ P_\infty B = \dfrac{v_2 - v_1 + \sqrt{v_1^2 + v_2^2}}{2}, \end{cases}$$

so that, using the cosine law in ΔAOP_∞,

$$v_\infty^2 = v_1^2 + P_\infty A^2 - 2 v_1 P_\infty A \cos\theta \quad \text{(see Fig.)}$$

$$\cos\theta = \frac{v_1}{\sqrt{v_1^2 + v_2^2}}, \quad \text{so that}$$

$$v_\infty^2 = v_1^2 + \frac{(v_1 - v_2)^2 + v_1^2 + v_2^2 + 2(v_1 - v_2)\sqrt{v_1^2 + v_2^2}}{4} - v_1\left(v_1 - v_2 + \sqrt{v_1^2 + v_2^2}\right) \cdot \frac{v_1}{\sqrt{v_1^2 + v_2^2}}$$

$$\Leftrightarrow v_\infty^2 = \frac{2v_1^2 - 2v_1 v_2 + 2v_2^2 + 2(v_1 - v_2)\sqrt{v_1^2 + v_2^2}}{4} - \frac{v_1^2 (v_1 - v_2)}{\sqrt{v_1^2 + v_2^2}}$$

$$= \frac{v_1^2 - v_1 v_2 + v_2^2}{2} + \frac{(v_2^2 - v_1^2)(v_1 - v_2)}{2\sqrt{v_1^2 + v_2^2}}$$

$$= \frac{v_1^2 - v_1 v_2 + v_2^2}{2} - \frac{(v_1 - v_2)^2 (v_1 + v_2)}{2\sqrt{v_1^2 + v_2^2}}$$

$$\Rightarrow \boxed{v_\infty = \sqrt{\frac{v_1^2 - v_1 v_2 + v_2^2}{2} - \frac{(v_1 - v_2)^2 (v_1 + v_2)}{2\sqrt{v_1^2 + v_2^2}}}} \quad (3)$$

a) · For $v_1 = v_2$, (3) leads to

$$\boxed{v_\infty = \frac{v}{\sqrt{2}}} = \frac{v_1}{\sqrt{2}} = \frac{v_2}{\sqrt{2}},$$

with $v = v_1 = v_2$. This is obvious due to symmetry: P must lie on the middle of AB, which is a distance $\frac{v}{\sqrt{2}}$ from the origin.

b) · In general, according to (3),

$$\boxed{v_\infty = \sqrt{\frac{v_1^2 - v_1 v_2 + v_2^2}{2} - \frac{(v_1 - v_2)^2 (v_1 + v_2)}{2\sqrt{v_1^2 + v_2^2}}}}$$

· ~~A~~ A limiting case is $v_1 = 0$ or $v_2 = 0$, in which case $v_\infty = 0$. The block does not start moving.