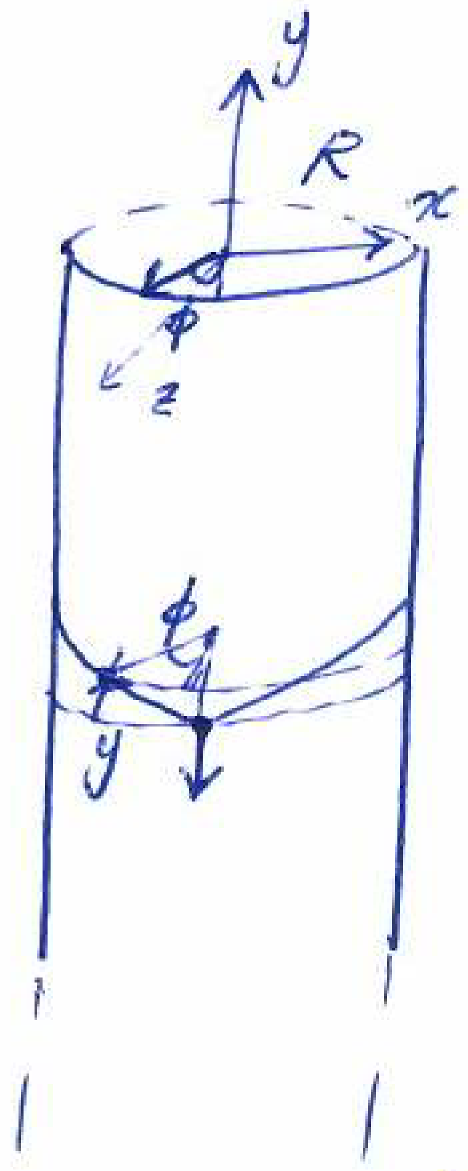
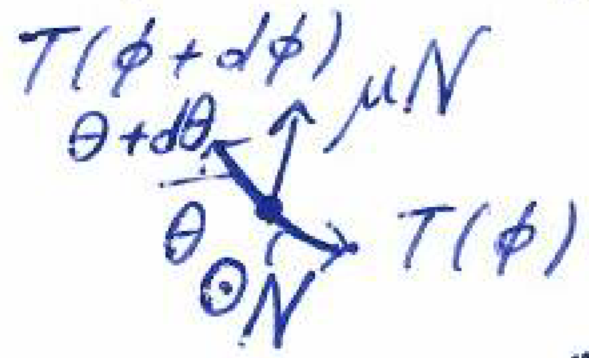
Emir Akdağ TUR 3



~~Let the y coordinate of the~~

Let y be the height of the portion of thread (wrt the free end of the thread).

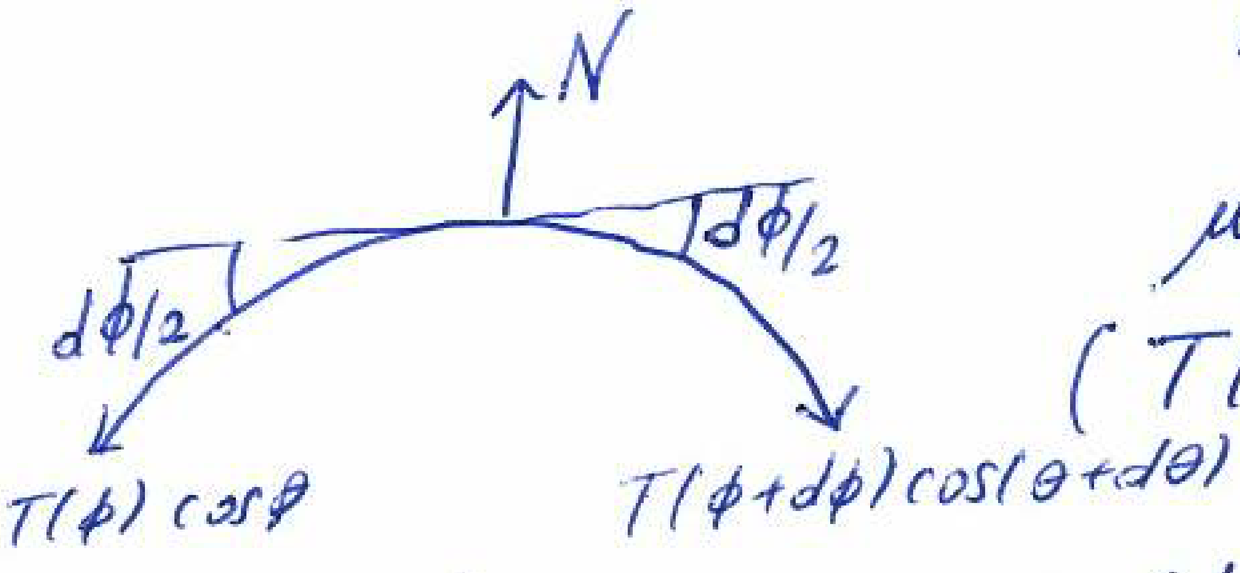


$$\tan\theta = \frac{dy}{R\,d\phi}$$

$$T(\phi+d\phi)\cos(\theta+d\theta) = T(\phi)\cos\theta$$

$$\mu N \geq T(\phi)\sin\theta - T(\phi+d\phi)\sin(\theta+d\theta)$$

$$\left(T(\phi+d\phi)\cos(\theta+d\theta) + T(\phi)\cos\theta\right)d\phi = 2N$$



$$d(T(\phi)\cos\theta) = 0$$

$$\Rightarrow N = T(\phi)\cos\theta\, d\phi$$

$$\mu T(\phi)\cos\theta\, d\phi \geq -d(T(\phi)\sin\theta)$$

$$T(\phi)\cos\theta \equiv T_0$$

$$\Rightarrow \mu T_0\, d\phi \geq -T_0\, d(\tan\theta) \quad \cancel{\Rightarrow \mu \geq -\frac{d^2y}{R\,d\phi^2}}$$

When the length of the loop is shorter than L_0 force ~~that is required to exert to the free end of the thread~~ that is exerted on the free end of the thread will increase the force of friction in return so it will never be possible to move the thread until the tension force reaches a critical value and breaks the thread.

Length of a small portion of the thread is $dl = \sqrt{(R\,d\phi)^2 + (dy)^2}$
$= R\,d\phi\sqrt{1+\tan^2\theta}$

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$$\frac{d\tan\phi}{d\phi} \leq -\mu \Rightarrow \frac{d\tan\theta}{d\phi} \geq \mu \Rightarrow \tan\theta \geq \mu\phi$$

$$dl = R\,d\phi\sqrt{1+\tan^2\theta} \geq R\,d\phi\sqrt{1+\mu^2\phi^2}$$

$$= \mu R\,d\phi\sqrt{1+\mu^2\phi^2}/\mu$$

$$L \geq \frac{1}{\mu} R \int_{\mu\pi} \sqrt{1+\mu^2\phi^2}\, d(\mu\phi)$$

$$= \frac{2R}{\mu}\int_0 dx\sqrt{1+x^2}, \quad x = \mu\phi$$

$\Rightarrow$

$$L \geq \frac{R}{\mu}\left(\mu\pi\sqrt{1+\mu^2\pi^2} + \ln\left(\mu\pi+\sqrt{1+\mu^2\pi^2}\right)\right)$$

$$\boxed{L_0 = \pi R\sqrt{1+\mu^2\pi^2} + \frac{R}{\mu}\ln\left(\mu\pi+\sqrt{1+\mu^2\pi^2}\right)}$$