## 1 Ice pellets

An interesting weather phenomenon can occur when the temperature profile in the atmosphere shows an inversion. The solid blue line in figure 1 shows such a temperature profile. The inversion occurs at heights between 1 km and 2 km.

Under these conditions snow falling through the atmosphere (partially) melts in the warmer layer and (partially) freezes again before reaching the ground in the form of "ice pellets".

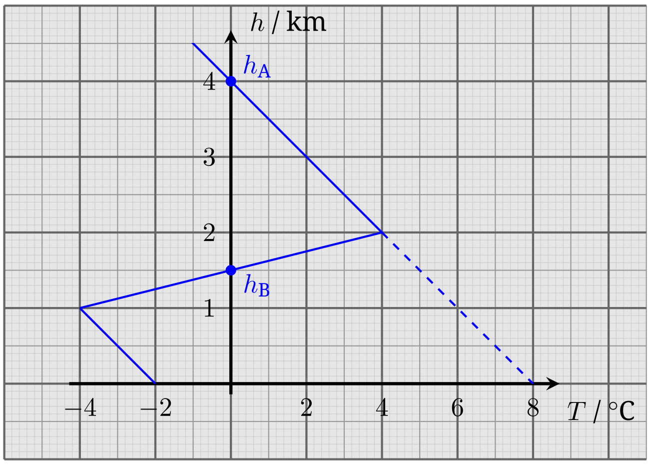


Figure 1: Atmospheric temperature $T$ vs. height $h$ above the ground.

Assume that a small, spherical ice droplet almost completely melts while falling through the atmospheric layer between $h_A$ and $h_B$ where the temperature is above freezing point.

a. Determine the mass fraction of the droplet that freezes before reaching the ground.

b. Find, as precisely as possible, the temperature of the droplet at ground level if there were no inversion and the temperature profile followed the dashed line below a height of 2 km.

Neglect evaporation, condensation and size changes of the droplet. Assume that water and ice have very high thermal conductivity and that the density of the atmosphere is constant with height.

Use $c_{\text{water}} = 4.2\,\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1}$ for the the specific heat of water and $c_{\text{ice}} = 2.1\,\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1}$ for that of ice. The specific latent heat for the melting of ice is $L = 334\,\text{kJ}\,\text{kg}^{-1}$.

## 2 Motion of a charged ball

A solid, homogeneous spherical ball of mass $m$ and radius $R$ is made of insulating material and has charge $Q$ distributed uniformly throughout its volume. The ball is placed on a large horizontal surface, and set in rolling motion without slipping in such a way that its center starts to move with initial horizontal velocity $v_0$. There is a uniform magnetic field (flux density) of magnitude $B$ perpendicular to the surface. The coefficient of static friction is large enough to prevent the ball from slipping on the surface. The moment of inertia of the ball about an axis through its center is $2mR^2/5$.

Describe the motion of the center of the ball and the shape of its trajectory.

Hint: Depending on your approach you may use the following identity:

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}\,(\vec{a} \cdot \vec{c}) - \vec{c}\,(\vec{a} \cdot \vec{b})$$

valid for any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

## 3 Water hose

A stream of water exits from the nozzle of a hose with a constant unknown speed $v$. A child plays with the hose by rotating it randomly in a fixed vertical $x$-$y$ plane. The nozzle is kept at $x = y = 0\,\text{m}$, and the angle between the nozzle's axis and the horizon is never less than $45°$. At each moment in time, the stream in the air has an irregular shape. The shape at one instant is shown in the figure below.

Using this figure, determine the exit speed $v$ if the free fall acceleration is $g = 9.8\,\text{m}/\text{s}^2$.

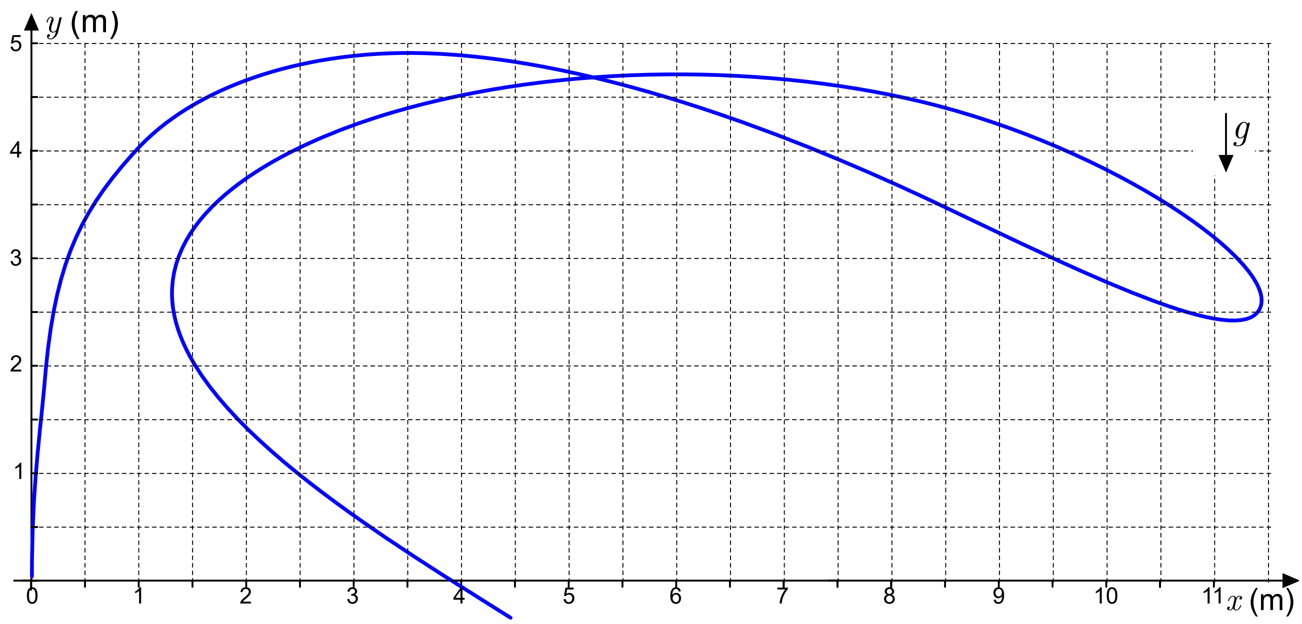


Figure 2: Shape of the water stream at a certain moment in time (larger version provided on separate sheet).